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HORIZONTAL DIFFUSION DUE TO OCEANIC TURBULENCE[1]

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ABSTRACT

The purpose of this paper is to discuss some fundamental aspects of horizontal diffusion in the sea and to present some observations that were made especially to obtain quantitive data.

The subject falls naturally into two parts:

Part I. It is shown that the classical Fickian equation of diffusion does not describe diffusion in the sea. The Richardson law of diffusion is introduced, and it is shown from observations of diffusion in the ocean that this new equation of diffusion does indeed describe the process.

Part II. Recent theories of turbulence are reviewed. The Weisaecker-Heisenberg theory of the spectrum of turbulence for large Reynolds number and the Kolmogoroff theory of locally isotropic turbulence are discussed. The applicability of these theories to oceanic turbulence is considered. It is shown that a 4/3 law for eddy viscosity is deducible from the theories of Weisaecker-Heisenberg and Kolmogoroff.

Part I seems safely applicable to the ocean because of the inductive nature of the theory. Part II, being deductive, contains certain assumptions and hypotheses which are discussed in some detail. Just how closely the Weisaecker-Heisenberg theory or the Kolmogoroff theory describes the turbulent regime in the ocean is open to question, but the fact that they both predict a 4/3 law, which is observed in certain ranges of scale, demands that they be seriously entertained.

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[2] Contribution No. 496 from the Woods Hole Oceanographic Institution.

I. INDUCTIVE METHOD

1. *Introduction.* The word "turbulence" is used in so many loose connections in oceanography that its meaning is often fuzzy and ill-defined. The dictionary definition of turbulence as "tempestuous state" or "great agitation" is obviously of little physical significance, because a physical quantity must be capable of measurement; that is, it must possess a numerical magnitude and dimensions. A fluid flow is said to be turbulent if it possesses a "turbulent velocity," the meaning of which arises in the following manner.

In laboratory experiments it is fairly easy to define the turbulent velocity exactly. For example, when air is drawn through a wind tunnel which is fitted with a coarse mesh screen at the entrance, the flow of air through the tunnel is complicated by large numbers of eddies set up by the screen. The size of these eddies immediately behind the screen is roughly the size of the mesh of the screen. In thinking of the velocity of the air at some fixed point within the wind tunnel it is a natural concept to regard this velocity as the sum of two terms: (1) a *basic velocity* which remains unchanged with time, or, in other words, the time mean velocity; (2) a fluctuating term which depends upon the eddies in the main stream and whose time mean vanishes, this term being called the *turbulent velocity.* Depending on the magnitude of the turbulent velocity, the flow can be said to be more or less turbulent, so that in this sense, and in this sense only, does the word turbulence make physical sense. In mathematical terms we may restate the foregoing remark as follows.

2. *Definition of Turbulent Velocity.* Suppose that at some fixed point the x-component of the fluid velocity is $u(t)$, a function of the time t. If the time mean of $u(t)$ is taken over a sufficiently long time T we approach a mean value U in the following manner,

$$U = \lim_{T \to \infty} \frac{1}{T} \int_0^T u(t)dt \,, \tag{1}$$

where U is called the *basic velocity.* The instantaneous velocity $u(t)$ is therefore expressible in the form

$$u(t) = U + \mathbf{u}(t) \,, \tag{2}$$

where, by definition of U,

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T \mathbf{u}(t)dt = 0 \,. \tag{3}$$

The additional term $\mathbf{u}(t)$ is called the x-component of the *turbulent velocity.* Similarly, the other components of instantaneous velocity

$v(t)$ and $w(t)$ may be expressed as the sums of component basic velocities V and W and component turbulent velocities $\mathbf{v}$ and $\mathbf{w}$.

The time mean square of the turbulent velocity,

$$\overline{\mathbf{u}^2} = \lim_{T \to \infty} \frac{1}{T} \int_0^T \mathbf{u}^2(t)dt \,, \tag{4}$$

does not vanish. The root time mean squares of the turbulent velocity components $\sqrt{\overline{\mathbf{u}^2}}$, $\sqrt{\overline{\mathbf{v}^2}}$, $\sqrt{\overline{\mathbf{w}^2}}$ are often called the component *intensities of turbulence.*

The *kinetic energy* of turbulence is defined as

$$\tfrac{1}{2}\rho\,(\overline{\mathbf{u}^2} + \overline{\mathbf{v}^2} + \overline{\mathbf{w}^2})\,.$$

These definitions show what is meant by turbulent velocity, intensity of turbulence, and energy of turbulence. It is also evident that these definitions depend entirely upon the premise that limits do exist as the value of T becomes very large. In the case of the wind tunnel experiment there is no question that the limits exist.

In large natural bodies of fluid, such as the ocean, it is by no means so obvious that limits do exist. If T is of the order of magnitude of one hour, then small scale turbulence, such as local mixing, will appear in the term $\mathbf{u}(t)$, whereas tidal currents and seasonal changes will occur in the basic velocity U. If a week is taken as T, then tidal currents will fall into the term $\mathbf{u}(t)$. If ten years is taken as T, then the seasonal changes will occur also in $\mathbf{u}(t)$. If a geological age is taken for T, even long range secular changes will appear in the turbulent velocity term $\mathbf{u}(t)$. Thus it is immediately obvious that in a large natural body of fluid the motion is vastly more complicated than in a well controlled wind tunnel. There are eddies of all sizes and velocities present, and it is by no means evident just what the basic velocity and what turbulent velocity should be called. This situation is remediable, however, as may be seen in the Weisaecker-Heisenberg theory. In the foregoing the averages have been time averages exclusively. It is equally possible to use space averages; in fact, we shall do so in the development of the Weisaecker-Heisenberg theory.

3. *The Fickian Equation of Diffusion.* In the Fickian equation of diffusion we assume that the limits of the integrals of equations (1), (3), and (4) exist. For simplicity of the exposition we shall limit ourselves to a study of the diffusion in the x-direction only.

If $\nu(x, t)$ is the concentration of a diffusing substance, and k is the molecular diffusivity, then the Fickian equation of molecular diffusion is

$$\frac{\partial \nu}{\partial t} + u(x, t)\frac{\partial \nu}{\partial x} = \frac{\partial}{\partial x}\left(k\frac{\partial \nu}{\partial x}\right). \tag{5}$$

Because of the difficulties of specifying $u(x, t)$, it is convenient to absorb the effect of the turbulent velocity upon the concentration into a so-called eddy diffusivity K. The Fickian equation of diffusivity is then written as

$$\frac{\partial \nu}{\partial t} + U\frac{\partial \nu}{\partial x} = \frac{\partial}{\partial x}\left(K\frac{\partial \nu}{\partial x}\right). \tag{6}$$

Equation (6) is applied frequently in oceanographic problems, and attempts are made to calculate the magnitude of K from observations of U and ν as functions of position and time. It is also possible to express K in terms of the turbulent velocity and a mixing length, following Prandtl. In oceanography it has become customary to think of K as an empirical constant. A knowledge of the eddy diffusivity alone is obviously not enough to determine the turbulent velocity, or the turbulent energy.

If the eddy diffusivity is taken as constant with x and $U = 0$, then the Fickian equation of diffusion (6) takes on the simple form

$$\frac{\partial \nu}{\partial t} = K\frac{\partial^2 \nu}{\partial x^2}. \tag{7}$$

A variety of integrals of this equation is available from the theory of heat conduction. From our point of view, perhaps the simplest is the solution for the diffusion of an instantaneous point source of an amount ν_0 of the diffusing substance, where x_1 is the point at which the instantaneous source was introduced. Since this diffusion is the result of the random motion of the diffusing particles, it is correct to regard the solution above as being indicative of the form of the curve describing the probable separation of a single particle from its starting point. In other words, the probability that a particle at x_1 at $t = 0$ will find itself at x at time t is

$$\frac{1}{\sqrt{4\pi K t}} \exp\left(\frac{-(x - x_1)^2}{4Kt}\right).$$

This notion of probability is important in the theory of diffusion in order to form an intuitive picture of what diffusion really means. It states, in effect, that the future probable position of a *particular* particle is independent of the concentration of neighboring particles $\nu(x)$ (or of $\partial\nu/\partial x$ or $\partial^2\nu/\partial x^2$ for that matter). The important idea to keep in mind is that in simple diffusion there is no "pressure" or "force" which tends

to push particles from regions of greater to regions of lesser concentration. A net flow of particles does occur in such a direction, but merely through the operation of random motion.

We shall now proceed to use this notion of probability for making measurements of the diffusion in the sea.

4. *Failure of the Fickian Equation of Diffusion.* It has been known for some time (Richardson, 1926) that the Fickian equation of diffusion does not apply to the diffusion of clusters in the atmosphere. The question of the applicability of the Fickian equation to diffusion in the ocean may be answered by the following test.

If two particles are placed at time $t = 0$, one at $x = 0$ and the other $x = b_0$, then at time t the probability of the first being at x and the second at $x + b_1$ is, respectively,

$$\frac{1}{\sqrt{4\pi Kt}} \exp\left(\frac{-x^2}{4Kt}\right),$$

and

$$\frac{1}{\sqrt{4\pi Kt}} \exp\left(-\frac{(x + b_1 - b_0)^2}{4Kt}\right).$$

These relations are simply integrals of the Fickian diffusion equation. The probability $P(b_0, b_1)$ that these two particles will be a distance b_1 apart at time t is the product of the two expressions above integrated over all x:

$$P(b_0, b_1) = \frac{1}{2\pi Kt} \exp\left(\frac{-(b_1 - b_0)^2}{4Kt}\right) \int_{-\infty}^{\infty} \exp$$

$$\left(-\frac{1}{Kt}(x^2 + x[b_1 - b_0])\right) dx = \frac{1}{2\sqrt{\pi Kt}} \exp\left(-\frac{(b_1 - b_0)^2}{4Kt}\right).$$

Therefore we see that the Fickian equation of diffusion leads to the result that the probability of a pair of particles a distance b_0 apart along the x-axis being a distance b_1 apart after an interval of time t depends upon $(b_1 - b_0)^2$ only and not upon either b_0 or b_1. The observations of Richardson and Stommel (1948) show that *this conclusion is at variance with the observed facts.* In the ocean the probability of large values of $(b_1 - b_0)^2$ actually increases greatly with b_0 and b_1. The data from the aerial photographs of floats at Woods Hole and Bermuda, discussed later in this paper, also exhibit this discrepancy. There appears to be *no way in which the Fickian diffusion law can be modified to meet this difficulty.*

Sverdrup (1946), in a consideration of the horizontal diffusion of dye spots in the ocean, has suggested that the Fickian equation of diffusion be used with the diffusivity as a linear function of the "average radius" of the spot. C. J. Burke (1946) has analyzed the diffusion of several dye spots over periods up to seven hours and has shown that Sverdrup's suggestion does provide a better fit than the assumption of any constant diffusivity. Although this suggestion is a useful expedient, it does not meet the fundamental difficulties. In a cloud of dye such as that envisaged by Sverdrup the diffusivity (even though it changes with the mean radius of the dye spot) is instantaneously the same everywhere in the cloud, for both close and wide pairs of particles. However, this is contrary to fact. The same objections seem to apply to O. G. Sutton's work (1932) in atmospheric diffusion.

5. *The Search for Another Law of Diffusion.* In molecular diffusion, which is successfully described by the Fickian equation of diffusion, the motion of each molecule is independent of that of its immediate neighbors; but in turbulent fluid flow, neighboring particles of fluids tend to have increasingly similar turbulent velocities as the distance between them is diminished. The reason for this is easily seen if a regime of turbulent eddies of all sizes is contemplated. The distance between two initially close particles will be changed at first only by the smallest eddies, the effect of the large eddies being simply to transport the pair as a whole while not tending to change their separation. However, when the separation of the two particles is larger, the large eddies, in addition to the small ones, can act to change the separation, so that as the separation is increased the dispersing influence of larger eddies is brought into play.

It is important to recognize, when thinking of a turbulent regime of eddies of all sizes, that the meaning of the term eddy is not to be restricted to a simple rotary motion of a certain mass of liquid. The term eddy is frequently used in this limited sense, as may be seen by such terms as "clockwise eddy," the implication being a rotary motion of the fluid in a clockwise sense.

Eddies, although they may be simple rotary motions of a fluid, need not be so. The essential idea of an eddy, as conceived in this paper, is as follows: In the neighborhood of a point of fluid A the particles at a distance α from A will have turbulent velocities to some degree similar to the turbulent velocity of A, and this similarity or local correlation of turbulent velocity extends to some distance $\alpha = \alpha_1$, the quantity α_1 then being thought of as the radius of the eddy.

Richardson (1926) saw that this essential difference between molecular diffusion and the diffusion in a turbulent regime necessitated a

complete reformulation of the equation of diffusion. The important independent variable was not position of a particle but its separation from its neighbors. This necessitated expressing the concentration of a diffusing substance as a function of the mutual separation of the particles of the diffusing substance, not as a function of position. Guided by the form of the Fickian equation, he postulated a new law of diffusion in which the term corresponding to the diffusivity in the Fickian equation was a function of mutual separation.

6. *The Richardson Equation of Diffusion.* The concentration of diffusing substance as a function of position is $\nu(x)$. This may be construed as meaning that there are $\nu(x)dx$ particles of the diffusing substance between x and $x + dx$. If we denote the separation between two particles by l, calling this the "neighbor separation" for convenience, then we can denote the number of particles which have neighbors with neighbor separations of between l and $l + dl$ as $q(l)dl$. The quantity $q(l)$ may be called the "neighbor concentration" in analogy to the concentration. Thus we may express neighbor concentration as a function of the mutual separation of particles rather than as a function of the position of the particles. The mathematical transformation from $\nu(x)$ to $q(l)$ is accomplished in the following way:

$$q(l) = \int_{-\infty}^{\infty} \nu(x)\, \nu(x + l)\, dx\,.$$

The equation of diffusion which Richardson postulated is

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial l}\left[F(l)\,\frac{\partial q}{\partial l}\right],$$

which is analogous to the Fickian equation, but with neighbor separation replacing position as the independent variable. The quantity $F(l)$ is analogous to the diffusivity K of the Fickian equation and may be called the "neighbor diffusivity" in order to distinguish it from the ordinary diffusivity of Fick. Because $F(l)$ is a function of l, it is possible to reconcile it with the observed facts. This, indeed, is the whole reason for the transformation from x to l as independent variable.

From a large number of observations with different values of l in the atmosphere, Richardson (1926) induced that the neighbor diffusivity is of the form

$$F(l) = \varepsilon l^{4/3}$$

in the air, where ε is a constant. This equation will be spoken of frequently as the "4/3 law."

The observations of Richardson and Stommel (1948) and the data from aerial photographs of floats at Woods Hole and Bermuda suggest that this law is generally operative in the diffusion of material in the ocean.

Although both neighbor diffusivity $F(l)$ and eddy diffusivity K have the same dimensions, it is clear that in general they are in no direct way comparable, for both are functions of different variables and are defined by different equations. However, in certain special cases it is possible to argue from the magnitude of one to that of the other. A discussion of this comparison is given in some detail in Richardson's (1926) paper. In the special case where $F(l)$ is independent of l, $F = 2K$. In the special case where the 4/3 law holds, that is, where $F(l) = \varepsilon l^{4/3}$, it is possible to get a rough idea of the comparison of K. Then $F(l) \backsimeq 3.03\ K$.

7. *A Method for the Determination of the Neighbor Diffusivity as a Function of the Neighbor Separation.* In a previous section we saw that the Fickian diffusion equation predicted that the probable change of separation of two floats did not depend upon the separation itself. This result was contrary to observed fact. We shall now make use of observations of this sort to indicate a method for determining the neighbor diffusivity as a function of the neighbor separation.

If floats are released in pairs at a fixed initial separation l_0 apart, and if the separation l_1 is measured again after a time interval T which is small enough so that $l_1 - l_0$ averages only a small fraction of l_0, it is seen that the scale will be practically the same for the observation of each pair. The Richardson equation of diffusion may then be written in the following form:

$$\frac{\partial q}{\partial t} = F(l_0)\,\frac{\partial^2 q}{\partial l^2}\,.$$

A solution of this equation is

$$q(l_1) = \frac{\text{const.}}{\sqrt{T}} \exp\left[-\frac{(l_1 - l_0)^2}{4TF(l_0)}\right],$$

which shows the distribution of $q(l_1)$ about the mean separation at time T. Of course at time $t = 0$ the entire population is at a separation l_0. The standard deviation of l_1, from the mean l_0, is $\sqrt{2TF(l_0)}$; therefore the neighbor diffusivity is given for the value l_0 by the following equation, where the bar denotes mean of all pairs:

$$F(l_0) = \frac{\overline{(l_1 - l_0)^2}}{2T}\,.$$

In practice the l_0 for each pair will not be identical, so it will be necessary to compute a mean of the type $\frac{1}{2}(\overline{l_1 + l_0})$ as a measure of the scale.

$$F(\tfrac{1}{2}(\overline{l_1 + l_0})) = \frac{\overline{(l_1 - l_0)^2}}{2T}$$

This equation is the one used in the reduction of the data observed in this paper. Many pairs were dropped into the water at different places simultaneously, so that from the photograph the various pairs are chosen at random in groups of approximately equal $\frac{1}{2}(l_1 + l_0)$. The quantities $\overline{\frac{1}{2}(l_1 + l_0)}$ and $\overline{(l_1 - l_0)^2}/2T$ are computed for each of these groups separately, each group giving a value of $F[\frac{1}{2}(\overline{l_1 + l_0})]$ for a particular neighbor separation.

8. *Methods of Observation at Sea.* The observations of Richardson and Stommel were very restricted in range of scale and were made in only one Scottish Loch. For universal application to the ocean these observations need to be repeated widely, and the range of scales must be greatly extended to include much greater distances. In the following paragraphs suggestions are made for methods of observation on various scales.

9. *The Scale from 10 M. to 1 Km.* Aerial photography of suitable floats answers best in this range, the floats being so submerged as to avoid the direct stress of the wind.

A number of ideas suggested themselves to this end. At first the use of fluorescein dye in water soluble form, distributed over the ocean surface in spots, seemed to be best. There could be little doubt that the motion of the dye and the water would be identical—at least in the horizontal direction. Two methods were used for dispersing the dye: (1) dropping it in cardboard ice cream containers which burst upon striking the water, leaving easily discernible splotches, and (2) introducing the dye directly into the ocean with a hose from a slow-moving surface ship. The greatest difficulty with the dye spots was judging the centers, errors of judgment in this respect being particularly bad for determination of neighbor diffusivity for small neighbor separations. In addition, the spots of dye themselves diffused and became indistinct so rapidly that they were no longer measurable in the time required for appreciable motions of wide pairs. This placed an upper limit on the neighbor separation as well.

It then occurred to the writer to try meteorological balloons, brightly colored and inflated with sea water instead of air. Enough fresh water could be added to balance the weight of the rubber itself so that

Figure 1. Two batches of paper floats diffusing on the sea surface. The boat gives a measure of the scale of the phenomenon.

the balloons floated just below the surface, the wind thus having little effect upon them. They were so heavy, however, that it was necessary to inflate them in the sea (handling them with fish-nets), and since the pumps available did not have a great capacity it took too long to inflate enough balloons to make a profitable series of photographs. However, the balloons were clearly visible and possibly the technique could be used for other purposes.

The most promising type of float was suggested by Dr. R. B. Montgomery—namely pieces of paper. The writer tried various kinds of paper from a skiff, and it appears that standard mimeograph paper answers well, this being sufficiently absorbent to become quickly wetted yet light enough to float. Several hundred sheets of such paper may be dropped from the airplane at an altitude of a hundred feet; they immediately scatter in the air, and upon striking the water surface they make a well defined pattern of points which may be photographed. Fig. 1 shows such a pattern of papers scattered on the sea surface.

10. *The Scale from 1 Km. to 1000 Km.* Large scales might be investigated by the use of long range sound transmission techniques in places where the detection stations are already established. Small floats, containing from two to five explosive pressure detonated charges which are dropped successively, could be set at wide distances apart on the sea surface from steamers in the regular lanes. The positions of the explosions might then be determined by the standard triangulation procedure.

The bombs could be released by salt blocks, a small compensating float being released with each bomb to preserve the trim of the remaining apparatus.

11. *General Remarks on the Observation of Floating Pairs.* Since the turbulent exchange is to be determined as a function of the scale of the phenomenon, it is important that the time interval between successive observations of the same pair be small enough so that the scale is not significantly altered between observations. On the other hand, the time interval should be large enough to avoid the influence of the observational errors in the distance measurements. The time interval of 30 seconds, employed in the observations of Richardson and Stommel (1948), was rather too large. A rough indication of the time intervals desirable for different scales may be obtained from the assumption that the preceding theory is correct. The following table was computed in this fashion and should give an indication of the time intervals that are likely to be useful.

Scale l_0	*Time interval between successive measurements, T*
10 cm	5 sec
1 m	25 sec
10 m	2 min
100 m	7 min
1 km	40 min
10 km	3 hr
100 km	12 hr
1000 km	3 days

A careful examination of the errors of observation must be made for any method of observation adopted, since random errors will have the effect of increasing the value of the turbulent exchange.

12. *Observations: Blairmore, Woods Hole, Bermuda. Blairmore*— The first observations made in the ocean for the purpose of testing the 4/3 law were those of Richardson and Stommel (1948), quoted here: "In the sea we used floats of parsnip because it is easily visible, and because it is almost completely immersed so as not to catch the wind which, moreover, was slight. The floats were about 2 cm in diameter. An optical device was used for measuring the distance l in a fixed azimuth. The observations were made in latitude 56° 0′ N, longitude 4° 54′ W from Blairmore Pier, Loch Long, Scotland, on 6 January 1948, where the sea water was about two meters deep. In order to eliminate any change in $F(l)$ with time, we observed alternately with large and small l. The function $F(l)$ was computed separately for the wide and close pairs":

	Wide pairs	*Close pairs*	*Unit*
l	187.7	26.7	cm
$F(l)$	84.3	6.4	$cm^2\ sec^{-1}$

Woods Hole—A number of observations of the diffusion of floats were made in Vineyard Sound near Woods Hole. Some of these were not successful for a variety of reasons, such as difficulty with the camera or an insufficient number of floats. Of the half dozen series of observations, two are chosen as being fairly representative.

Series 1: Dye spots were introduced into the water by means of a hose from a slow-moving boat. A wooden frame-work was floated in the water so that it would also appear in the photographs and thus provide an accurate scale for each picture. The depth of water was about 25 m, and a strong tidal current prevailed. The time interval was 2 min.

Close pairs:		*Wide pairs:*	
l_0	l_1	l_0	l_1
cm	cm	cm	cm
1450	1040	9500	8500
1540	1670	9200	8400
1805	1670	9500	6900
2450	1990	7400	8600
1370	1350	11600	10350
2140	1840		
2120	2150		
2120	2150		
1820	2130		
1660	920		
2320	1990		
1820	1680		

In addition to the observations of the motion of pairs of dye spots, there was the diffusion of the dye in the spots themselves which gave a rough approximation of the diffusion on even a smaller scale. For this purpose one may compute the ordinary diffusivity K and then convert it to $F(l)$ by multiplying by 3.03, although admittedly this is not a completely satisfactory or strictly logical procedure. In this way the following table was computed:

	Wide pairs	*Close pairs*	*Diffusion of a single spot*	*Units*
l	9000	1800	290	cm
$F(l)$	2400	120	50	$cm^2\ sec^{-1}$

Series 2: Pieces of mimeograph paper were dropped from an airplane in Vineyard Sound off the Falmouth Beach in about 4 to 10 meters of water. There was a very strong tidal current. From the air, clouds of sediment, stirred up from the bottom, could be seen. The time interval was 80 secs.

Close pairs:		*Wide pairs:*	
l_0	l_1	l_0	l_1
cm	cm	cm	cm
61	102	466	353
87	22	466	493
105	29	484	210
61	61	448	492
79	61	353	529
87	51	475	598
123	95	448	407
123	73	456	368

Close pairs:		*Wide pairs:*	
l_0	l_1	l_0	l_1
cm	cm	cm	cm
96	44	456	456
87	44	353	370
87	80		
52	86		
87	44		
96	87		
87	114		

The reduction of these observations leads to the following values:

	Wide pairs	*Close pairs*	*Unit*
l	430	77	cm
$F(l)$	92	10.4	$cm^2\ sec^{-1}$

Bermuda—A typical series of measurements from aerial photographs off Bermuda in deep water is given in the following table. The water was about 2,000 meters deep.

Five Miles SE of Cataract Hill, Bermuda, November 22, 1948

Close pairs:		*Wide pairs:*	
l_0	l_1	l_0	l_1
cm	cm	cm	cm
111	91	600	585
66	80	840	695
66	128	870	951
45	131	690	805
69	146	810	329
45	0	810	768
81	201	1050	1207
75	110	990	1299
114	146	540	480
84	219	810	1006
150	36	750	860
150	201	915	256

The reduction of these measurements leads to the following values:

	Wide pairs	*Close pairs*	*Unit*
l	795	103	cm
$F(l)$	337	36	$cm^2\ sec^{-1}$

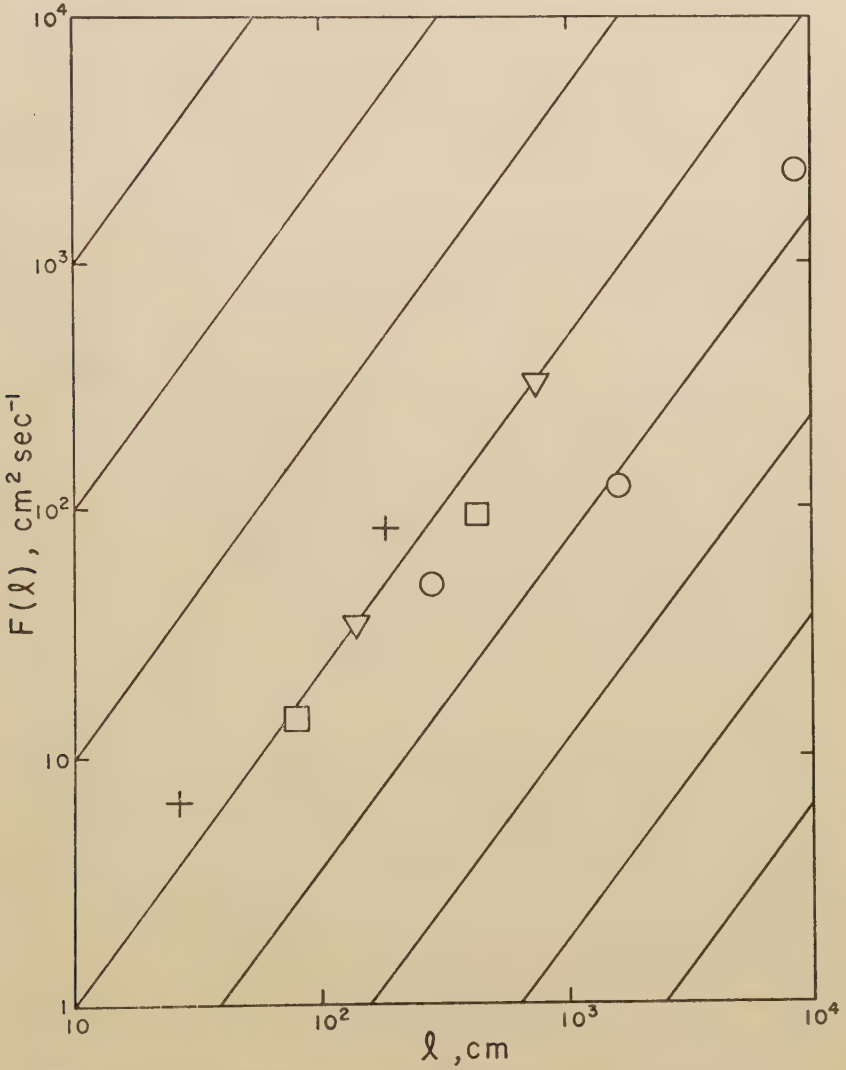

Figure 2. Data plotted for measurements given in the text. The crosses are for data obtained at Blairmore; circles, Woods Hole, Series 1; squares, Woods Hole, series 2; triangles, Bermuda.

13. *Summary of Observations.* The values of the neighbor diffusivity are plotted against the neighbor separation (for the observations listed) in Fig. 2. Lines are drawn on these graphs to show the direction of

the 4/3 slope. The constant of proportionality ϵ is not the same for all the graphs.

The data plotted seem to justify the supposition that the 4/3 law is satisfied. If the classical Fickian law were obeyed, the points for any particular set of observations would have equal $F(l)$ instead of an increase with l.

II. DEDUCTIVE METHODS

14. *Introduction.* The observational methods of studying ocean turbulence, as described in the first part of this paper, were based upon an attempt to find a law of diffusion which fitted the observed facts. The method involved a number of theoretical ideas, all inductive in nature. The result of this process was the rejection of the Fickian law of diffusion and the introduction of the Richardson law. The observations confirm the discovery that in the ocean, as well as in the atmosphere, the neighbor diffusivity of fluid particles is proportional to the 4/3 power of the neighbor separation measured along a fixed azimuth.

We now turn to a completely different mode of study, in which the energy decay of large scale motions of a fluid is examined, the method being a deductive one based on von Weisaecker (1948) and Heisenberg (1948). The remarkable result emerges that in turbulent regimes of large Reynolds number the eddy viscosity obeys a closely analogous 4/3 law. This deduced result is entirely independent of the previous inductive approach.

15. *The Necessity of a Spectrum of Eddies in the Ocean.* The semi-permanent wind systems, such as the trades and the prevailing westerlies, impart large scale motions to the ocean surface layers by virtue of the frictional stress of the wind acting upon the water. The source of energy of the large scale ocean currents, such as the Equatorial Currents, is explained in this way. The stress of the wind, working upon the ocean water, tends to increase its kinetic energy. However, the mean kinetic energy of the ocean currents remains fairly constant, so that on the whole as much energy must be dissipated into heat as is gained from the work done by the wind stress. Molecular viscosity alone is insufficient to dissipate enough kinetic energy to act as an effective brake upon the major ocean currents. However, ocean currents break up into eddies, which in turn degenerate into smaller eddies, and so on, until eddies are formed that are of a size sufficiently small to be dissipated irreversibly by molecular viscosity into heat energy. The exact dynamical explanation of the cause of the breakdown of the ocean currents into eddies of various sizes is yet to be

formulated, but the empirical fact of their existence is not disputed (Iselin and Fuglister, 1948; Spilhaus, 1940). The existence of an entire "spectrum" of eddies is apparently necessary as an agency capable of dissipating the kinetic energy of the ocean currents that is supplied by the stress of the wind.

The completely detailed field of motion in the ocean is therefore exceedingly complicated, constantly changing with time and defying minute description in much the same way as the individual molecular motions of a liter of air.

The eddy motion of fluids in flumes and wind tunnels has been treated in a statistical fashion (Taylor, 1935, 1938; Dryden, 1943), but until recently there has been no similar attack upon the problem of the large scale motions of the atmosphere and ocean.

In practical oceanography it has long been recognized that the term "velocity," when applied to an ocean current, always involves a mean over a certain time interval or volume, or both, depending on the method of observation employed. For example, the determination of currents from a compilation of the drift of ships involves an averaging of velocity over at least several hours and scores of miles. Another oceanographic example is the determination of a velocity field by the standard method of dynamic computations, in which the spacing of the hydrographic stations defines the scale of the averaging; the velocities obtained are averages over the distance between successive stations. A picture of the velocities determined in this manner lacks the finer scale features which are present. Therefore, it is important to recognize that *the term "velocity" is meaningless unless accompanied by some indication of the mode of averaging employed in its determination.*

Once the mode of averaging has been decided, the velocity field has a meaning. The components of water motion that occur on a scale smaller than that used in the velocity averaging process do not appear in this "average velocity field," but they do exert an influence upon the dynamics of the average velocity field as Reynolds stresses. The Reynolds stress involves a quantity known as the eddy viscosity which depends upon the mode of averaging. In other words, *the term eddy viscosity is meaningless unless accompanied by some indication of the mode of averaging employed in its determination.*

16. *Recent Theoretical Studies of Turbulence at Large Reynolds Number.* During the war years the fundamental nature of turbulence was temporarily neglected. However, three remarkable papers appeared quite independently which shed light on the spectrum of turbulence at large Reynolds numbers. The first of these was by A. N. Kolmogoroff (1941) of Russia; in the original form, these Kolmogoroff papers were

extremely condensed, so that Batchelor (1947) was led to discuss the theory at some length to make it available to investigators in general. The second paper was by L. Onsager (1945) in the United States. The third in this remarkable series was by the German astrophysicist, C. F. von Weisaecker (1948), whose ideas have been developed and elaborated by Werner Heisenberg (1948). With certain modifications of a minor kind, such as those indicated by the writer, these theories may prove to be valuable in getting at a clearer picture of the spectral distribution of energy in the turbulence of the ocean. First we shall introduce certain hypotheses about the eddies in the ocean which lead to the Weisaecker-Heisenberg development. Then we shall reexamine these hypotheses in the light of actual facts—*so far as we know them*—to see how they must be modified for a closer fit to things as they are. This latter process will be particularly valuable because it will point out where our empirical ignorance lies and where observational material is desperately needed.

17. *The Weisaecker-Heisenberg Theory.* Suppose that in the ocean we could take as premises:

Hypothesis 1. There exists a continuous series of eddies of all sizes which are horizontally isotropic; that is, the quadratic time mean square of the velocity is independent of choice of co-ordinate direction.

Hypothesis 2. A constant supply of kinetic energy is available to the large eddies (ocean currents driven by wind stress).

Hypothesis 3. This energy sifts down to smaller eddies.

Hypothesis 4. The smallest eddies lose their energy into heat energy.

Can we now discover, on the basis of these hypotheses, from the equations of viscous flow and continuity:

(1) How the energy is distributed over eddies of different size (that is, the spectrum of turbulence)?

(2) In what manner the average velocity is dependent upon the mode of averaging?

(3) In what manner the eddy viscosity is dependent upon the mode of averaging?

The work of Weisaecker and Heisenberg makes it possible to give answers to these questions provided an additional hypothesis is introduced, the nature of which will be shown when it is introduced.

In this section we adapt the Weisaecker argument to lateral turbulence in the sea, where it is more natural (except for the smallest scales) to average over surface areas than over volumes. We shall need to average over a wide variety of size areas.

The largest area may be taken as a square of side L_0. Suppose that this area is divided into smaller squares of side L_1, where $L_0 = rL_1$.

Each of these squares is divided into smaller squares of side L_2, and so on, the process being repeated according to the formula

$$L_n = rL_{n+1}\,, \tag{1}$$

where r is some positive integer independent of n. The scale—finer as n increases—is represented by the quantity L_n, and an average over a square of side L_n may be spoken of as the n-th mode of averaging.

Let the x-component of the velocity be u and the y-component be v. We may divide these velocities into a sum of terms such as follows:

$$\begin{aligned} u &= u_0 + u_1 + u_2 + \ldots \\ v &= v_0 + v_1 + v_2 + \ldots, \end{aligned} \tag{2}$$

with the subscript quantities defined as follows:

$$\begin{aligned} u_0 &= \text{average of } u \text{ over square } L_0 \\ u_1 &= \text{average of } (u - u_0) \text{ over square } L_1 \\ u_{n+1} &= \text{average of } (u - U_n) \text{ over square } L_{n+1} \end{aligned} \tag{3}$$

where $U_n = \sum_{m=0}^{m=n} u_m$. Since u_n is a function of time with a definite root time mean square, this is denoted by

$$\overline{U_n} = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_0^T (U_n)^2\, dt}\,. \tag{4}$$

It is convenient to define another quantity, $\mathbf{u}_n$, in the following manner:

$$\mathbf{u}_n = \sum_{m=n+1}^{m=8} u_n\,. \tag{5}$$

The total velocity u may then be written

$$u = U_n + \mathbf{u}_n\,, \tag{6}$$

where, with respect to the n-th mode of averaging, U_n is the average velocity and $\mathbf{u}_n$ the turbulent components. Depending upon the particular mode of averaging (value of n), the values of U_n and $\mathbf{u}_n$ are different. If we introduce time averages as in equation (4), the equation (2) may be rewritten in the following manner:

$$\bar{u} = \overline{U_n} + \overline{\mathbf{u}_n}\,. \tag{7}$$

In a similar manner averages of the derivatives may be denoted

$$\frac{\partial u}{\partial x} = \left(\frac{\partial u}{\partial x}\right)_0 + \left(\frac{\partial u}{\partial x}\right)_1 + \ldots\,. \tag{8}$$

We denote

$$\overline{u_n'} = \sqrt{\lim_{T\to\infty} \int_0^T \left(\frac{\partial u}{\partial x}\right)_n^2 dt}\,. \tag{9}$$

It is necessary to discover some relation between $\overline{u_n}$ and $\overline{u_n'}$, and for this purpose we write the formal relation

$$\overline{u_n'} = \alpha \overline{u_n}/L_n\,. \tag{10}$$

We now find it necessary to introduce an hypothesis:

Hypothesis 5: The eddy motion is kinematically similar; this requires that all statistical characteristics of turbulence are independent of the absolute linear dimensions.

Therefore, α is independent of n.

The energy dissipation per unit time and unit volume is

$$S = \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right], \tag{11}$$

in which μ is the molecular viscosity. If μ_n is introduced as the time average turbulent exchange, equation (11) may still be used. The turbulent exchange μ_n depends upon $\overline{\mathbf{u}_n}$ and may be written formally in the way Prandtl introduced it,

$$\mu_n = \rho\, l_n\, \overline{\mathbf{u}_n}\,, \tag{12}$$

where l_n is the "mixing length" of the n-th mode of averaging. The quantity l_n is defined in terms of L_n by

$$l_n = \beta L_n\,, \tag{13}$$

where, by Hypothesis 5, β is independent of n. Because of the isotropy introduced by Hypothesis 1, equation (11) becomes

$$S_n = \mu_n\, 8 \sum_{m=0}^{m=n} (\overline{u_m'})^2. \tag{14}$$

Now, S_n must be independent of n because the flow of energy cannot depend upon a division into average and turbulent flow. Quite formally we write

$$u_{n+1} = \xi\, u_n\,, \tag{15}$$

where $\xi < 1$ and is independent of n by Hypothesis 5. Therefore, $\overline{\mathbf{u}_n}$ may be replaced by the following identity:

$$\overline{\mathbf{u}_n} = \frac{\xi}{1-\xi}\,\overline{u_n}\,. \tag{16}$$

Substituting from equation (10) into (14),

$$S_n = 8\mu_n \alpha^2 \sum_{m=0}^{m=n} \left(\frac{\overline{u_m}}{L_m}\right)^2, \tag{17}$$

and by virtue of (15) this may be summed to

$$S_n = 8\mu_n \alpha^2 \left(\frac{\overline{u_n}}{L_n}\right)^2 \frac{(\xi r)^2}{(\xi r)^2 - 1}, \tag{18}$$

and from equations (12), (13) and (16)

$$S = \Gamma \frac{\overline{u_n}^3}{L_n}, \tag{19}$$

where

$$\Gamma = 8\rho\beta\alpha^2 \frac{\xi}{1-\xi} \frac{(\xi r)^2}{(\xi r)^2 - 1} \tag{20}$$

is independent of n. This is von Weisaecker's result. Therefore the spectral law for velocity is the following:

$$\overline{u_n} \propto L_n^{1/3}. \tag{21}$$

The spectral law for eddy viscosity is as follows:

$$\mu_n \propto L_n^{4/3}. \tag{22}$$

The spectral law for energy is as follows:

$$\overline{E_n} \propto L_n^{2/3}. \tag{23}$$

In discussing the spectral laws, von Weisaecker used a discrete spectrum, but Heisenberg has elaborated this technique and extended it to cover a continuous series of modes of averaging.

18. *Critique of the Applicability of the Weisaecker-Heisenberg Theory to the Ocean.* It is important to examine the hypotheses of the Weisaecker-Heisenberg theory in the light of how they bear upon the application of the theory to the turbulence in the ocean.

First of all, the semipermanent wind systems, though of primary importance, are not the only cause of water motion. Local winds supply energy to smaller scale eddy systems, and so do local tidal currents. In addition, thermal convection in the surface layers may play a role at certain levels of the eddy spectrum. These secondary sources of kinetic energy prevent the dissipation S_n from being entirely independent of n, rather making S_n increase with n. To the extent that these effects enter, Hypotheses 2 and 3 must be altered.

Hypothesis 1 is certainly not true for the real ocean. That there is a spectrum of eddies is not disputed, but the supposition that they are horizontally isotropic is open to question, particularly for the large size eddies. Certainly for the largest eddies—the major ocean currents—there is no isotropy at all. If the initial mode of averaging $n = 0$ is taken, so that L_0^2 is approximately the area of the ocean (for example, the North Atlantic Ocean), it is clear that for $n = 0, 1, 2$, up to some value $n = p$ at least, Hypothesis 1 cannot apply. Whether Hypothesis 1 applies in the actual ocean for n greater than p is a question that must be answered by observation. This is really a result in itself, because it points out the need for and the nature of certain types of current observation at sea. We may venture to introduce an alternate hypothesis which is more likely to be applicable to the ocean but which is as yet not thoroughly tested by observation.

Hypothesis 1′: The series of eddies in the ocean is horizontally isotropic for modes of averaging for which n is greater than p, but not necessarily so for $n \leqslant p$.

The Hypotheses 2, 3, and 4 apply to the ocean as it actually is. Hypothesis 5 must be replaced by another in the following fashion.

Hypothesis 5′: The eddy motion is kinematically similar for $n > p$.

On the basis of these slight changes in hypotheses, we introduce

$$\overline{\mathcal{U}_p} = \overline{u_0} + \overline{u_1} + \ldots \overline{u_p}\,, \tag{24}$$

where $\mathcal{U}_p$ is the sum of the large size averaging components of velocity which do not fulfill the restrictions of isotropy and kinematic similarity. Equation (2) may then be written in the following form:

$$\bar{u} = \overline{\mathcal{U}_p} + \overline{u_{p+1}} + \overline{u_{p+2}} + \ldots \quad . \tag{25}$$

Quantities such as $\overline{U_n}$ and $\overline{\mathbf{u}_n}$ still retain their meanings, but the summation processes cannot be carried out for all n. The independence of α and β of n holds only for those $n > p$. It will be helpful also to define another quantity $\overline{U}_{p,n}$ similar to $\overline{U}_n$ but in which the range of summation extends from $m = p + 1$ to $m = n$,

$$\overline{U_{p,\,n}} = \sum_{m=p+1}^{m=n} \overline{u_m}\,. \tag{26}$$

Therefore,

$$\bar{u} = \overline{\mathcal{U}_p} + \overline{U_{p,n}} + \overline{\mathbf{u}_n}\,. \tag{27}$$

This equation illustrates clearly the meanings of the various terms from a physical point of view. The rms (quadratic time mean square)

point velocity is regarded as being made up of three parts, each of which is a sum over a certain range of the spectrum of turbulence. The first term represents the sum of the rms velocities of the anisotropic, kinematically dissimilar largest scale horizontal motions; the second term is the sum of the rms velocities of the isotropic, kinematically similar horizontal eddies down to and including the n-th mode of averaging; the third term is the sum of the eddy rms velocities smaller than n-th mode of averaging. The first two terms are, from the point of view of the n-th mode of averaging, the "average velocity" of the fluid and the third term contains the turbulent fluctuations of the velocity.

19. *Kolmogoroff's Theory of Locally Isotropic Turbulence: The Eddy Cascade.* The theory of Weisaecker (1948) and Heisenberg (1948) was anticipated by A. N. Kolmogoroff (1941) of Russia, but the form in which it was written was analytically quite different, so that the immediate connection of Kolmogoroff's work with the type of observation on diffusion of clusters described in Part I was not evident to the writer. It was only after the writer had seen the manuscript of Weisaecker that he saw the connection; then later, when he came across the paper of Batchelor (1947), the fact that Kolmogoroff had been treating the same problem was made clear.

Kolmogoroff treats the correlations of the differences of parallel velocity fluctuations at two points in a certain domain G in which the turbulence is isotropic and statistically steady. This is insured by choice of domain G and by placing an upper bound upon r, the distance between the two points. Now, supposing that the Reynolds number of the flow as a whole is very large (as it is in natural bodies of fluid like the ocean), Kolmogoroff introduces two similarity hypotheses:

(1) The statistical characteristics of the turbulent flow are functions of the molecular viscosity and the mean energy dissipation per unit mass of the fluid only.

(2) The statistical characteristics of the large eddies depend upon the mean dissipation of energy per unit mass of the fluid only.

Batchelor (1947) finds that these two hypotheses hold reasonably well in the turbulence of a wind tunnel. The observations given in Part I of this paper seem to indicate that the hypotheses hold for the ocean as well.

From these hypotheses Kolmogoroff then proceeded to show by a dimensional argument that the double parallel velocity correlation is of the form $1 - Ar^{2/3}$, where A is a constant and r is limited to a certain middle range. This equation is similar to Heisenberg's (1948) equation 57.

20. *Onsager's Theory of the "Violet Catastrophe."*[3] The theory of L. Onsager (1945), published in abstract only, has much in common with the other two theories. It, too, deals with turbulent flow in which the Reynolds number of the entire motion is indefinitely large, in which molecular viscosity has little influence except near $r = 0$, in which the energy of each size eddy comes from larger ones and is dissipated into smaller ones until the smallest eddies are reached, which are laminar and dissipate their energy into heat via molecular viscosity. Onsager starts by describing the turbulent regime in terms of three-dimensional Fourier series. From the equations of mean motion he is able to determine the transfer of kinetic energy from one wave-length velocity component to another, determining the distribution of energy with wave-length. He then uses the Fourier transform to obtain (Taylor, 1938) the correlation coefficient which agrees in form with that found by Kolmogoroff.

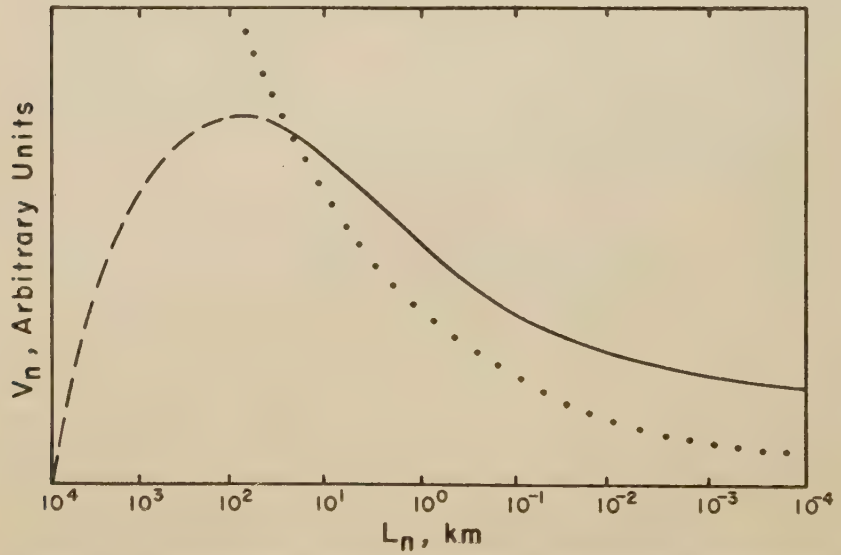

Figure 3. The solid line is a hypothetical spectrum of the root mean square turbulent velocity in the ocean. The dashed portion on the curve is the more uncertain portion of the hypothetical spectrum. The dotted line is the pure Weisaecker-Heisenberg theoretical spectrum.

21. *Hypothetical Spectrum of Turbulence in the Deep Ocean.* Presumably the velocity spectrum of turbulence at a point in the deep

[3] So-called because all kinetic energy degenerates toward the short end of the spectrum.

ocean will look something like Fig. 3. The scale of the ordinates is linear but in arbitrary units, a constant factor being indeterminate until observed values of the intermediate scales are available. The left-hand portion of the curve is dashed to indicate that it is particularly uncertain due to the anisotropy and kinematic dissimilarity it exhibits. It is important to remember, in considering this hypothetical picture, that the Weisaecker-Heisenberg theory applies only to that part of turbulent energy descending from large scale systems. The additional effects of local winds, bottom friction, and of irregularities in the bottom, are not taken into account in the Weisaecker-Heisenberg approach, which, after all, is hardly to be expected. Inshore localities and tideways exhibit turbulence of a local nature, but even this turbulence, in the middle range, appears to follow the 4/3 law, as is shown by the Blairmore and Woods Hole observations.

The right-hand portion of the curve is drawn with slightly less negative slope than the Weisaecker-Heisenberg theory in order to indicate that energy is also received by the ocean from sources other (local winds) than descent from the large scale motions.

22. *Summary.* As explained in the prefatory abstract, this paper represents a synthesis of two independent methods of approach in the study of turbulence and turbulent diffusion as applied to the ocean. The first part of this paper, dealing with inductive methods, stands quite by itself, supported as it is by observation. It represents a marked departure from ordinary notions of diffusion in the sea. The second part of this paper, summarizing the deductive theories of locally isotropic turbulence, is more abstract, less substantiated by observation; however, since it fits so well as a possible mechanism which explains the somewhat anomalous behavior of a turbulent regime as induced from the first part, it seems well worth considering when we turn attention to the details of the turbulent motion which is producing the diffusion in question.

It is always tempting in preparing a techical paper to refrain from discussing unsettled questions, in this way giving the finished product a polished appearance. Had this been the goal, the first part alone would have been enough by itself, with a new law of diffusion for the sea and observations to support it. But one inevitably inquires as to the "why" of this new law of diffusion, and in so doing must refer to the nature of the turbulent regime itself. This naturally leads to questions about the various scales of turbulence in the ocean dealt with in the second part of this paper. By a happy coincidence, investigators in other branches of fluid mechanics have already begun to study the spectrum of turbulence at large Reynolds number, or as it is altern-

atively called, the "eddy cascade" and the "violet catastrophe." The immediate applicability of these notions to the sea is controversial, but oceanographers should consider them seriously.

And finally, what is the value of studies such as these, other than any practical application of the diffusion law? The writer thinks there is a question of morale. At present oceanographers recognize that there is no perfectly simple circulation in the ocean, that currents are not clearly defined rivers in the sea, and that even the Gulf Stream wanders and meanders, so that the more closely one looks at it the more bewildering it is. In fact, there seems to be no means of achieving a sharply focused picture in the study of the ocean currents. What appeared yesterday as a smooth broad stream appears today as an amazingly corrugated irregular one, and tomorrow probably even finer details of greater complexity will make an appearance. So the oceanographer is assailed by growing doubts as to whether or not he can ever hope to describe the details with any meaning. The Weisaecker-Heisenberg theory shows in principle, at least, how such a description can be made in the language of the statistical theory of turbulence. Therefore it is a useful tool in thinking about the ocean's motion.

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A STREAMLINE CABLE DEPRESSOR

By

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In the past the collection of samples from the deep scattering layer by the familiar horizontal net haul method has been handicapped by the available tools. The time-honored weighted line technique has often been suspected of introducing error in the depth factor of data. The development of the streamlined cable depressor herein described provides an effective tool for oceanographic studies. Because of its form, the depressor has a high lift/drag ratio and is free from the damaging vibration usually characteristic of depressors. Many applications are suggested by its performance in the field.

Experimental hauls made by members of the Scripps Institution of Oceanography and of the Oceanographic Studies Section of the U. S. Navy Electronics Laboratory have given evidence of the effectiveness of this cable depressor as a tool capable of maintaining stable towing depth. The use of recording depth gages provides valuable evidence for records.

Fig. 1 shows a series of three depth records made while using a No. 1 depressor at slow speeds. The depth gage used in these tests had an original maximum of 400 feet, but it has been modified to give a 1,067-foot full scale range. The notation of "2M. NET" refers to a conical specimen gathering net assembled by LaFond and Tucker for the initial depressor tests. The diameter is two meters at the open end, the length 25 feet, the mesh two inches.

Chart No. I shows that the pen was off scale, riding against the stop pin for 30 minutes with 1,300 meters of cable out. With 1,000 meters of cable, the depth was decreased to 1,100 feet and thereafter it responded to each decreased cable length.

Chart No. II gives good verification of No. I. The one meter net was of fine mesh, which had greater drag than the longer two meter net with larger mesh.

Chart No. III shows operation at about 800 feet of depth with the two meter net attached. The ship course was reversed after runs I and II, which apparently changed the ship/water current relationship. It is evident that the depth was both stable and controllable. A

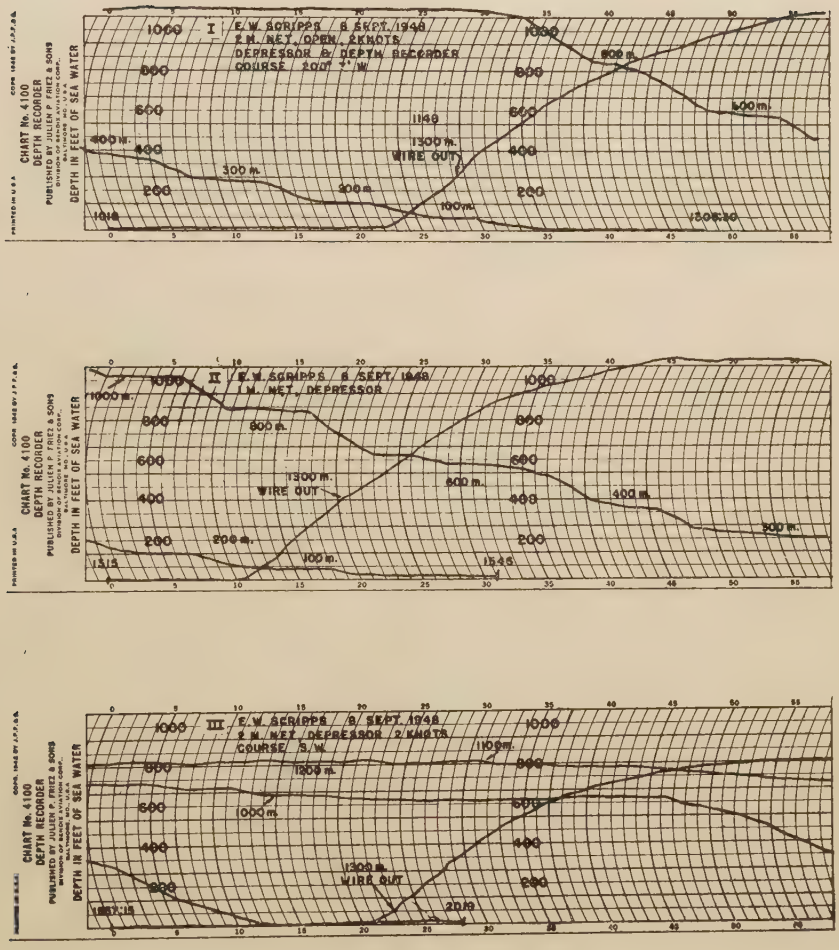


Figure 1. Depth Recorder Charts No. I, II, and III, Recorded by U. S. Navy Electronics Laboratory.

series of charts (three shown in Fig. 1) furnished by the NEL Oceanographic Studies Section shows over 20 hours towing at depths between 600 and 1,200 feet.

The successful deep sea operation at low ship speed was gratifying, since it proved to be an unexpected depressor characteristic, the design having been based on higher speed performance.

Judging from the interest shown in both the depressor and the

associated cable fairing by M. B. Schaefer of the Fish and Wildlife Service, it is likely that others may find interest in the data on operation at 5 to 15 knots.

Desired Performance Characteristics. The design of the NEL Depressor is based solely on its desired performance characteristics, which are: (a) High lift to drag ratio at all speeds; (b) freedom from vibration (self induced); (c) inherent towing stability; (d) freedom from protruding parts; (e) positive buoyancy; (f) minimum size and weight in light of (a) to (e).

Developed Design and Size. The developed depressor form results from the intersection of its functional surfaces, which may be described as follows. The upper surface is formed by two "triangular" planes which join along the centerline of the body (see Fig. 2). The planes are each inclined downward from the centerline at 25 degrees from horizontal. The lower surfaces have the general curvature of an airfoil. This surface is also divided at the centerline and each half has a downward angle of five degrees from the horizontal. The intersection of the upper planes and lower curved surface thus create a closed space, which is the body form. The curvature of the lower surfaces is developed by the joining of two conic sections and one cylindrical section, Fig. 2 (1). The two conic sections create the nose curvature, their base radii determining the nose shape of the center rib, their common apex locating the wing tip. The large radius is that of a cylindrical section which intersects the upper plane and smoothly joins the nose cone, thus forming the major wing surface. The "wing tips" are positioned to produce a sweepback of 16 degrees.

The photographs in Fig. 3 illustrate the appearance of the depressor. Four graduated sizes are suspended in the relative position they assume in operation. These are numbered in development sequence from No. 1, the largest, to No. 4, the last and smallest. Numbers 1 and 3 have proven most useful.

The construction of the depressors shown in the photograph is the same. The body is constructed of mahogany, built up of ¾-inch stock. The ends of the dowels which connect the two wing sections can be seen in the No. 4 depressor. Eight of the dowels pass through the dural centerline rib. The twelve dowels along the trailing edge (6 visible) reinforce the thin edges and do not extend through the rib. The indicated structure meets the requirements (d), (e), and (f). Lead inserts weighing five pounds were placed in the nose of the No. 3 depressor to aid in meeting the stability requirements (c). The depressor body has a fineness ratio of 6 : 1 at the centerline. The finish is smooth varnish.

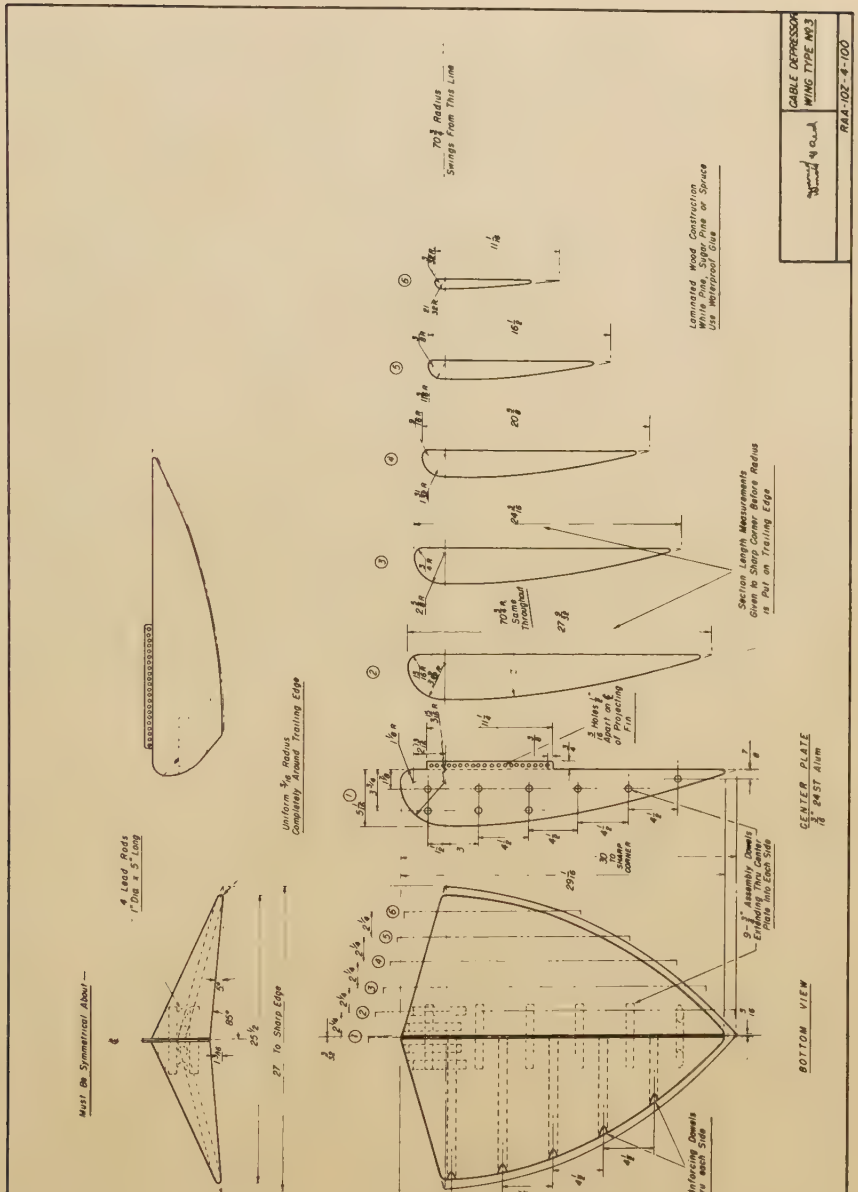
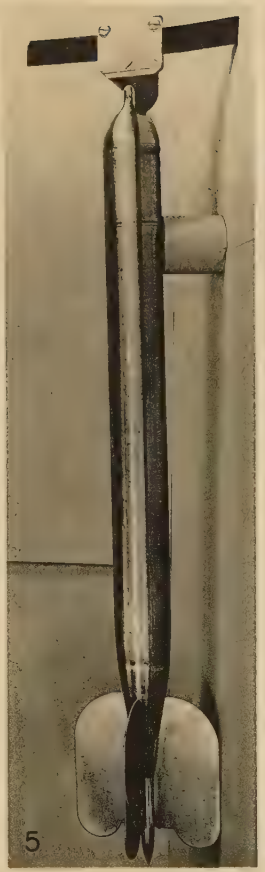


Figure 2 – Mechanical drawing of depressor.



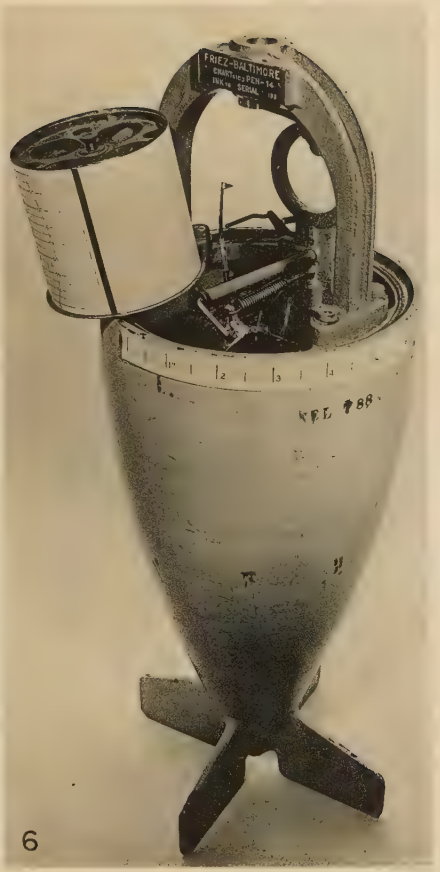


Figure 3 – Photographs of four sizes of depressor. Figure 4 – Water speedometer for small craft. Figure 5 – Inclinometer. Figure 6 – David Taylor Model Recording Depth Gage.

Explanation of Performance and Stability Characteristics. A simplified description of the forces acting on the surfaces of the depressor is as follows.

(a) The depressor is attached at the end of its tow line by its yoke, or bridle, in a single point suspension. It is free to pivot on this one point in any direction. The attachment is located above a point approximately one-quarter of the total length distant from the nose.

(b) When the tow line is put in motion the body of the depressor aligns itself with the direction of motion. Because the nose is loaded with lead, the body floats tail high, with a large attack angle. Forward motion establishes the conditions which create both negative lift and stability.

(c) The lower surfaces have the general curvature of an airfoil, as shown in Fig. 2 (1). The upper surfaces are plane. The planing effect of the inclined surfaces as well as the pressure decrease created by the increased water velocity as it passes under the body create the negative lift of the depressor. The center of lift is well forward of the dimensional center of the midrib.

(d) In stable operation, the top of the depressor is inclined nose down approximately nine degrees from the horizontal. As the body moves forward, the forward projected area of the two upper surfaces is not large. The nine degree attack angle in combination with the large dihedral give two planes obliquely inclined against the common center line. The flow of water over these planes creates forces which are symmetrical and produce no resulting moment as long as the direction of motion of the depressor coincides with the direction of the line of tow. If the depressor direction is changed by angular rotation about the center of lift, then restoring moments result and the body returns to a stable condition of tow.

(e) Assume for illustrative purposes that the depressor has a position of lateral displacement from a vertical plane through the line of motion of the tow-craft. In this position the body attitude will be that in which one wing (foil) is higher than the other because the direction of force in this towline must pass approximately through the center of lift. In this position the nose, loaded by lead slugs, causes the body to pivot nose downward about its center of lift. When the depressor reaches a point in the vertical plane through the line of motion of the tow craft, the heavy nose, by reason of being at its lowest possible position, ceases to supply a torque and a condition of equilibrium exists.

Performance of the depressor has met the desired stability requirements within limits herein described. It is recommended that any

use of the depressors be limited to speeds of 5 to 15 knots. There is no indication of depressor vibration under any circumstance.

Observed Operational Characteristics. An effort has been made to determine the lift/drag ratio of the depressor by means of measurements taken during towing operations in calm water. The measurements taken were: (a) Tow craft speed; (b) cable tension; (c) cable angle at water surface, vertical reference; (d) cable angle at depressor attachment, vertical reference; (e) depressor top chord angle, vertical reference; (f) depressor depth in feet; (g) cable arc, vertical reference; (h) cable length.

The instruments and methods employed in making the measurements were:

(a) A special water speedometer, Fig. 4. This device is a modified bicycle speedometer, driven through a torque tube and bevel gears by an impellor. Two impellors were used, one for 0 to 15 knots, the second for 0 to 30 knots. The pitch of the impellors was calculated to drive the speedometer at its normal RPM/mile rate to avoid exceeding its design characteristics. The calibration was accomplished by constant speed runs over a measured mile course.

(b) Cable tension was determined by using a Dillon Dynamometer, which was calibrated against a secondary standard.

(c) Cable angle at surface was determined by direct observation with level and protractor.

(d) Cable angles below water surface were determined by a special inclinometer (see Fig. 5).

(e) Depressor top chord angle was measured by towing close to the surface and by direct observation as in item (c).

(f) All depth measurements were made by use of a modified recording depth gage, DTMB model, made by Friez Instruments, Fig. 6. The instrument was given static check calibration before and after each day's operation.

(g) Cable arc was determined by incremental measurement. The measurements were taken along the cable by use of the inclinometer and recording depth gage at 25-foot intervals, starting three feet above the depressor. The observed depths and tangent angles permitted plotting of the cable curvature. Fig. 7 shows these curves, which are applicable to the speeds noted, and serves to indicate the variation in depressor depth with speed for any length of faired cable up to 500 feet. For example, at a speed of 5 knots, taking Y as the distance (in feet) of the depressor behind the tow craft and X as the depth of the depressor (in feet):

$$Y = .00106X^2 + .301X\,.$$

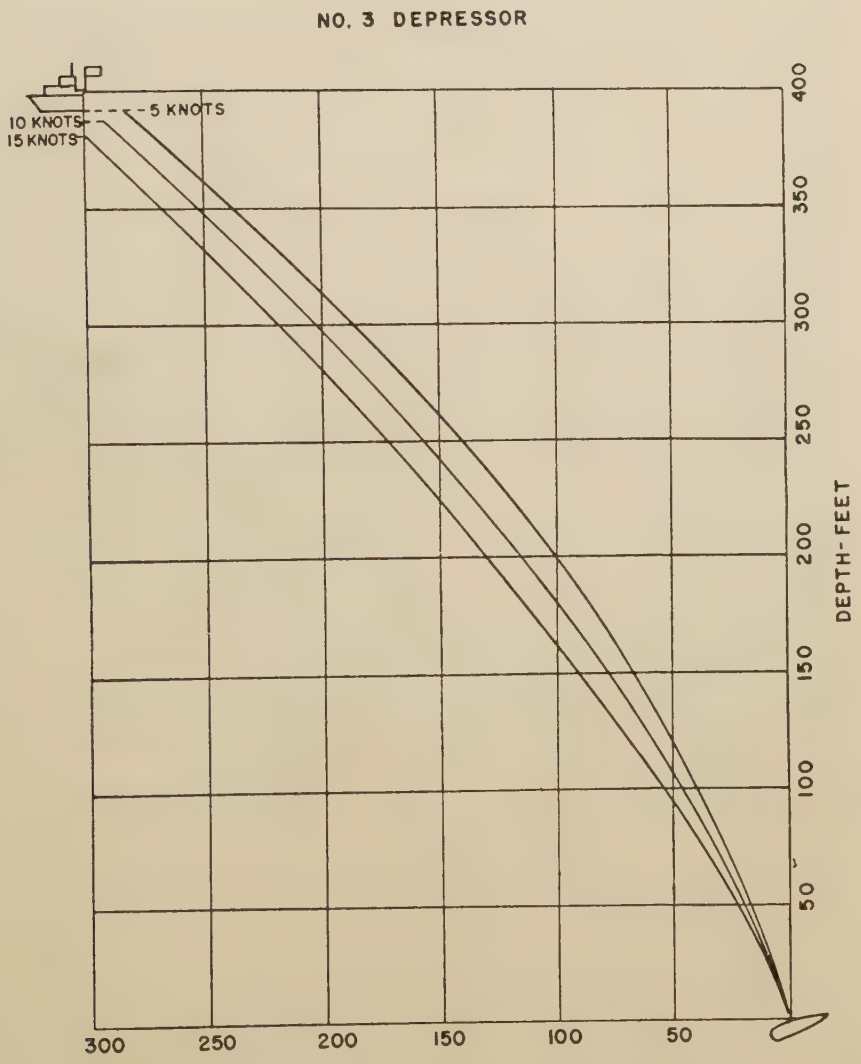


Figure 7. Cable Curvature Chart—500 ft. Cable.

A fair approximation of the cable length required for a desired depth may be obtained by scaling the graph. If greater precision is needed, the following equation will serve:

$$\text{Cable length (feet)} = \frac{PQ - b\sqrt{a}}{N^2} + \frac{R}{N^3} \log \frac{NQ + P}{N\sqrt{a} + b},$$

where $N = .00425$, $R = .0000180$, $P = 2Cx - b$, $Q = (a + b + xc^2)$ ½, x = depth (feet), $a = 1.09$, $b = .00128$, $c = .00000451$.

(h) Cable length was measured without tension. The stretch of the cable under load was not included.

The negative lift developed by the depressor was measured by simple instruments. The depressor was towed in calm water on a short (10 ft.) length of faired cable. The cable tension (S) was read directly from a dynamometer and the cable angle measured with level and protractor. If the angular displacement (θ) of the tow cable from vertical is assumed to be caused by depressor drag (neglecting the short cable) then:

$$S \sin \theta = \text{drag, and}$$

$$S \cos \theta = \text{lift.}$$

The No. 3 depressor moving at a speed of 15 knots develops a cable tension of about 1,500 pounds and an angle θ of 12 degrees. This gives a lift/drag ratio of 1,470/300, approximately 5/1. As the No. 3 depressor has a projected plan area of about 3.5 sq. ft., the wing loading is approximately 400 lb./ft.^{2} at the 15 knot speed.

The thrust-speed chart, Fig. 8, shows the thrust which may be expected for two depressors, Sizes 1 and 3. These curves are for the case of minimum stable attack angle. The maximum stable thrust which may be attained with the depressors is not yet known.

The position of the tow cable attachment determines the attack angle. The approximate midpoint of the attachment is located at a point about 26 % of the centerline chord measured from the leading edge. Incremental adjustment from the midpoint is in 1/10-inch steps. The maximum total movement in either direction will not exceed ½-inch.

To obtain the depth vs. cable length vs. speed performance indicated by Fig. 7, it is necessary to cover the tow cable with fairing. The function of the fairing is to reduce the turbulence about the cable and thereby reduce the parasitic drag on it. Several cable fairings, which materially reduce the parasitic drag, have been tested by this activity.

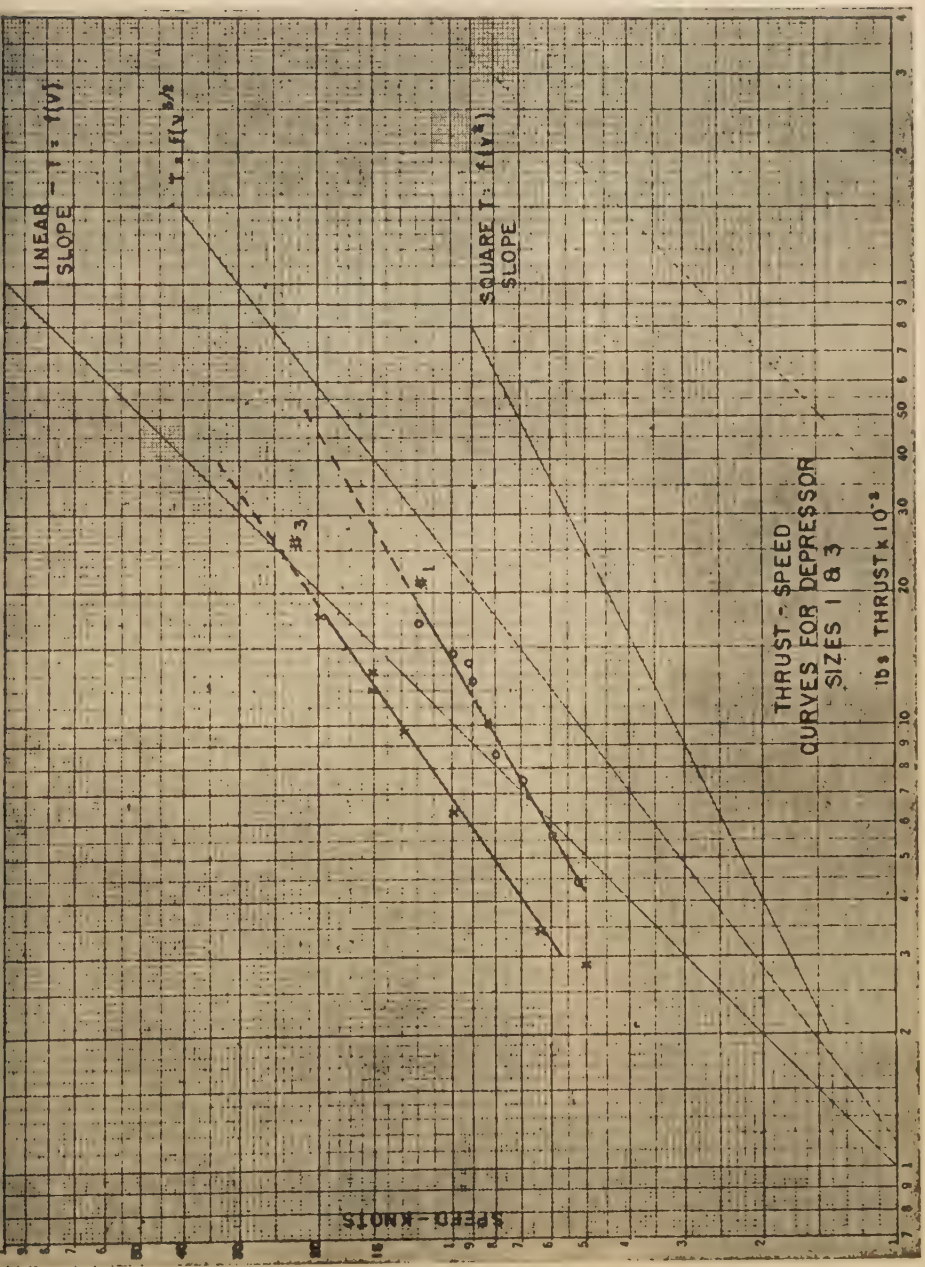


Figure 8. Thrust-Speed Charts for No. 1 and 3 Depressors.

Fig. 9 shows the form of a fairing developed by Navy Electronics Laboratory. NEL tests indicate a drag of 1.1 lb./ft. at 10 knots for this fairing in a condition of normal flow. The fairing was used in tests illustrated in Fig. 7.

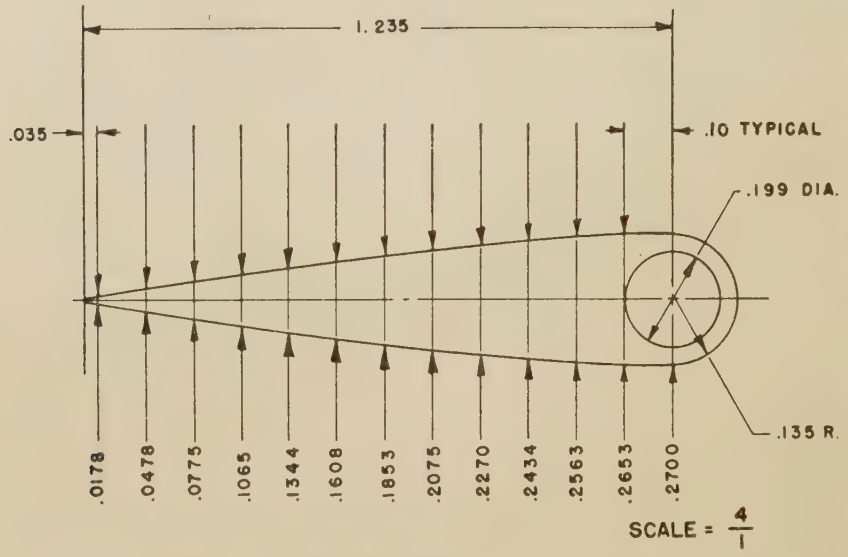


Figure 9. USNEL Cable Fairing Dimensions and Form.

Conclusions and Recommendations. It is apparent that the NEL depressor is an effective tool, and that many applications will be found for its use.

In launching a depressor it is necessary to pay out several hundred feet of cable, with the depressor and other gear attached before starting the ship in motion. Bare headway should be maintained until the depressor has reached stable depth, as a fast start will cause the depressor to dive at high speed and thereby develop damaging cable tension.

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